Modeling and Control of Fuel to Air Ratio in an Internal Combustion Engine with Unknown Evaporation Time and Fuel Fraction Impinging Manifold’s Wall

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Abstract
In this paper we develop a model for fuel to air ratio (FAR) in internal combustion engine (I.C.E). Based on this model, we can represent the FAR in terms of the fuel injected to the manifold, evaporation time, and the part of fuel that impinges the wall. This model is derived in this paper from other models existing in the literature in (I.C.E). The importance of this proposed model is represented in providing a controllable dynamics that allows us to design a control law for achieving a desired FAR in spite of unknown evaporation time and injected fuel fraction impinging the wall. This usefulness of the model is demonstrated in this paper throughout designing a control law to make the FAR tracks a desired value using Routh-Herwitze criterion. Both of the proposed model and control law are proved mathematically. Moreover, a numerical simulation is performed and shows the validity of the proposed model and designed controller.

Nomenclature

- FAR: Fuel-to-Air Ratio
- $\tau_{\text{ff}}$: Time constant of fuel film evaporation (s).
- $n$: Fraction of injected fuel.
- $m_{\text{ff}}$: Mass of intake fuel film (kg).
- $\nu$: Mass flow rate (kg/s).
- $a, b$: Negative and positive constants respectively

1 Introduction
In the last decade, increasing attention has been devoted to the study of transient phenomena related to the operation and the design of Engine Control Systems (ECS) to design suitable compensation strategies for mixture strength excursions and to assure good conversion levels for pollutant gases in catalytic converters. In order to accomplish exhaust regulations, an accurate control of Air fuel ratio has to be
guaranteed both in steady state and transient conditions, estimating correctly the air mass flow at the injector location and injecting the fuel in the right amount and with the appropriate time dependence. This goal can be reached making use of engine control strategies based on both online measurements and dynamic model estimations. Many engine dynamic models have been described in literature, characterized by different structures, goals and complexity. They range from very detailed fluid dynamic models [Low, et al., 1981], utilized for engine development, to black-box models mainly used for control system design [Powell and Powers, 1981].

They allow to describe the dynamics of both the air and the fuel flows making use of the mean values of the most important engine variables, and can achieve a good level of accuracy within a wide range of engine operating conditions, with a limited computational demand which make them particularly suitable for the design of the next generation of model based engine control strategies [Hendricks et al., 1992]. Many of the current production fuel-injection controllers rely on open-loop feed-forward control based on a look-up table with PI (proportional plus integral) feedback control [Canale, et al., 1978]. Other linear control techniques, such as LQG, pole placement and LQG/LTR have little advantage, since they need the output magnitude information while the control system is nonlinear and the sensor output is nearly binary [Cassidy, et al., 1994]. As a solution to this problem, a sliding mode fuel injection control method was proposed [Cho, and Hedrick, 1988]. There has been a great deal of research on transient air/fuel characteristics, and it is concluded that three characteristic delays are responsible for unwanted air/fuel ratio excursions during the transient operations [Aquino, 1981]. Hendricks et al. suggested a Mean Values engine model and observation air/fuel control techniques using their Mean Value model [Baruah, 1990]. The local aspects of joint air and fuel control have been studied in [Stefanopoulou, et al., 1994] by designing a linear controller based around a specific operating point. Chang et al. suggested a similar event based observation control technique [Amstutz, Fekete, and Powell, 1994]. An adaptive compensation of the fuel dynamics by means of a fuzzy-based algorithm is suggested in [Moraal, 1995].

An online identification of the dynamics of AFR in injected engines and, therefore, a time varying model of the engine to be used for a subsequent adaptive control design is given in [Turin and Geering, 1993]. Choi and Hedrick developed an observer-based fuel injection control algorithm to improve the AFR control by using sliding mode control method [Choi.& Hendrick, 1998]. Yoon and Sunwoo use an adaptive dynamic sliding-mode control to deal with the problem caused by engine uncertainties [Yoon, 2001].

Manzie proposed a radial basis function (RBF) neural network based approach for the fuel injection control problem and found that this network is suitable for estimating the air mass flow into the cylinder [Manzie, et al., 2001]. The lean engines currently in production use the fuel consumption advantage only in steady-state operation and not during transients [Horie, et al., 1992]. Takagi and Sugeno [Takagi and Sugeno, 1985] introduced a different linguistic description of the output fuzzy sets, and a numerical optimization approach to design fuzzy controller structures. They range from detailed models, mainly oriented to facilitate the design of air/fuel ratio (AFR) control systems, describing the mixture formation phenomena [Yildiray, et al., 2009].

The main contribution of this paper we developed a dynamic equation model of a manifold fuel injection engine to reach FAR to the ideal value and design of stable controllers that achieve stability and convergence of its error signals to predefined error bounds for this dynamic model systems with the presence of the fuel injected by the injector hits the manifold wall and forms a thin film on it. by using Routh-Herwitz criterion.
In Section 2, we state the problem. In Section 3, we explain the main results and present the controllers structure; the proof of stability and convergence, and the robustness properties of the controllers for system with fuel hits the manifold wall. FAR control problem is also treated in section 4. Section 5 contains concluding remarks and recommendations for future work.

2 Problem statement

Fuel injector is a pressurized nozzle that sprays fuel into the incoming air on Spark Ignition (SI) engines as shown in figure (1). On SI engines fuel injectors are located at the intake valve ports on multipoint port injector systems and upstream at the intake manifold inlet on throttle body injector systems. A part of the fuel injected by the injector hits the manifold wall and forms a thin film on it. Moreover, fuel from the manifold fuel film continuously evaporates, and that leads to inhibit the FAR to reach the ideal value because of the fuel losses which take place by this process. The goal is to derive a mathematical model that describes accurately the evolution of fuel flow entering the cylinder, given fuel flow coming out of the fuel injector. For this reason we propose a mathematical model in order to describe an accurate FAR dynamics. The derivation of this proposed model given bellow is given in the main results section.

\[
\dot{x}_p = ax_p + b[v_1(t)[1 - n] + \left(\frac{m_{ff}(t)}{\tau_{ff}}\right)]
\]

\(x_p\) represents the measured FAR, a and b are negative and positive constants respectively, \(v_2\) is the mass flow rate of fuel delivered by the fuel injector, \(n\) is the fraction of the fuel that impinges on the wall, \(m_{ff}\) is the mass of the manifold fuel film. \([1 - n]\) is the non-impinging part of the injected fuel flow, and \(\tau_{ff}\) is time constant for the first order evaporation model.

Assumption 1:- \(\tau_{ff}\) and \(n\) are unknown parameters, that are usually assumed constants for a given operating conditions. Both \(n\) and \(\tau_{ff}\) are assumed bounded by known constants such that \((0 < n < 1)\) and \(\tau_1 \leq \tau_{ff} \leq \tau_2\), where \(\tau_i\) and \(\tau_2\) are known constants.

3 Main Results

The main results of this work will be presented in two theorems:

**Theorem 1:**
The fuel to air ratio of an internal combustion engine can be described by model (1).
Proof: To derive (1), consider the following reduced order plant model represented in state space form [Yildiray Yildiz, Anuradha M. Annaswamy Diana Yanakiev Ilya Kolmanovsky, 2009].

\[
\dot{x}_p = ax_p + bv
\]  

(2)

Where \(v\) is the net fuel flow entering the cylinder.

The fuel film dynamic model can be described by the following mathematical formula [Pushkaraj A. Panse, 2005]:

\[
v(t) = v_1(t)(1 - n) + \left(\frac{m_{ff}(t)}{\tau_{ff}}\right)
\]  

(3)

\[
m_{ff}(t) = v_1(t)n - \left(\frac{m_{ff}(t)}{\tau_{ff}}\right)
\]  

(4)

Assuming zero initial conditions, the Laplace transform for (3) & (4) yield:

\[
V(s) = V_1(s)(1 - n) + \frac{M_{ff}(s)}{\tau_{ff}}
\]  

(5)

\[
sM_{ff}(s) = nV_1(s) - \frac{M_{ff}(s)}{\tau_{ff}}
\]  

\[.: M_{ff}(s) = \frac{nV_1(s)}{s + \frac{1}{\tau_{ff}}}
\]  

(6)

Substituting (6) into (5), we obtain:

\[
V(s) = V_1(s)(1 - n) + \left(\frac{1}{\tau_{ff}}\right)nV_1(s)\left(s + \frac{1}{\tau_{ff}}\right)
\]

By rearranging this equation, we get:

\[
V(s) = V_1(s)(1 - n) + \frac{nV_1(s)}{s\tau_{ff} + 1}
\]

\[
V(s) = V_1(s)[(1 - n) + \frac{n}{s\tau_{ff} + 1}]
\]  

(7)

Substitution of (7) into the Laplace transform of (2) yields:

\[
sX_p = ax_p + b\{V_1(s)[(1 - n) + \frac{n}{s\tau_{ff} + 1}]\}
\]

\[
sX_p = ax_p + b\{V_1(s)(1 - n) + \frac{nV_1(s)}{s\tau_{ff} + 1}\}
\]  

(8)

From (6) and (8), we find:

\[
sX_p = ax_p + b\{V_1(s)(1 - n) + \frac{M_{ff}}{\tau_{ff}}\}
\]

(9)

The Laplace inverse of (9), gives (1). \(\square\)
Remark 1: The proposed model (1) can accurately describe the dynamics of FAR in I.C.E. with the property that it enables us to control the FAR throughout controlling the fuel injected to the manifold, while the reduced model (2) employed in [Yildiray et al., 2009] can control the FAR by using only the fuel exiting from the injector. Therefore, our new model demonstrates the effect of evaporation time and fuel fraction impinging the manifold's wall and consequently we can compensate for these effects throughout selecting an appropriate control law in this model. As we shall in the following theorem, this model enables us successfully to design a controller that achieves the desired FAR with presence of unknown evaporation time and fuel fraction impinging the manifold's wall.

THEOREM 2:
Under assumption 1, system (1) is stable if we satisfy the control law:

\[ v_1(t) = \frac{1}{b} \{ A \int_0^t e(\tau)d\tau + B \exp(-\lambda_1 t) \int_0^t e(\tau) \exp(\lambda_1 \tau)d\tau \\ + C \exp(-\lambda_2 t) \int_0^t e(\tau) \exp(\lambda_2 \tau)d\tau + D \exp(-\lambda_3 t) \int_0^t e(\tau) \exp(\lambda_3 \tau)d\tau \} \]

(1)

0)

With,

\[ A = \frac{-a}{\lambda_1 \lambda_2 \lambda_3}, \quad B = \frac{(-\lambda_1 - a)(-\lambda_1 + 1)}{-\lambda_1(-\lambda_1 + \lambda_2)(-\lambda_1 + \lambda_3)}, \quad C = \frac{(-\lambda_2 - a)(-\lambda_2 + 1)}{-\lambda_2(-\lambda_2 + \lambda_1)(-\lambda_2 + \lambda_3)}, \]

and

\[ D = \frac{(-\lambda_3 - a)(-\lambda_3 + 1)}{-\lambda_3(-\lambda_3 + \lambda_1)(-\lambda_3 + \lambda_2)} \]

Where \( \lambda_1 = -0.5 + \frac{\sqrt{7}}{2}i, \lambda_2 = -0.5 - \frac{\sqrt{7}}{2}i, \) and \( \lambda_3 = \frac{1}{\tau_{ff} - n\tau_{ff}} \).

Proof:
Let \( V_1(t) = EV_2(t) \)

From (11) and the Laplace transform of the controller (10), we find:

\[ V_2 = \frac{(s - a)(s\tau_{ff} + 1)}{b(s\tau_{ff} - ns\tau_{ff} + 1)} \left[ \frac{1}{s^3 + s^2 + 2s} \right] \]

(12)

Substituting (11) into the Laplace transform of (1), we are led to

\[ X_p = \frac{b}{s - a} \{ EV_2(s)(1 - n) + \frac{n}{s\tau_{ff} + 1} \} \]

\[ X_p = \frac{bEV_2(s)}{s - a} \left[ \frac{s\tau_{ff} - ns\tau_{ff} + 1}{s\tau_{ff} + 1} \right] \]

(13)

System (13) can be represented by the block diagram shown in figure (2)

Figure (2). Block Diagram of System (13)
System (13) can be reduced to a simple block diagram shown in figure (3).

![Block Diagram of System (13)](image)

From Fig.(3) we can say

\[ X(s) = E(s)G_1(s)G_2(s) \]
\[ X(s) = R(s) - E(s) \]
\[ R(s) - E(s) = E(s)G_1(s)G_2(s) \]
\[ E(s) = \frac{1}{1 + G_1(s)G_2(s)}R(s) \]  

(14)

From Fig.(2), the corresponding characteristic equation is:

\[ 1 + GH = 1 + \frac{(s - a)(s\tau_g + 1)}{b(s\tau_g - ns\tau_g - 1)} \frac{1}{s^3 + s^2 + 2s} \frac{b(s\tau_g - ns\tau_g + 1)}{(s - a)(s\tau_g + 1)} \]

\[ 1 + GH = 1 + \frac{1}{s^3 + s^2 + 2s} \]  

(15)

Equating (15) to zero, we find

\[ s^3 + s^2 + 2s + 1 = 0 \]

The corresponding Routh table is constructed as follows:

<table>
<thead>
<tr>
<th>s^3</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>s^2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>s^1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>s^0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Since, the closed-loop transfer function has all poles in the left half of the s-plane, the system is stable according to Routh-Hurwitz, such a condition can be guaranteed if there are no sign changes in the first column of the Routh table, and this is clearly satisfied from above Routh table.

### 4 Simulation

In this section, we demonstrate the validity of the model proposed in Theorem 1 and the controller designed in Theorem 2 throughout considering a case study of fuel/air ratio (FAR) control of an internal combustion engine (I.C.E.). Based on Theorem 1, the dynamics of this case study can be represented by the following equation:
\[ \dot{x}_p = ax_p + b[v_1(t)(1 - n) + \left(\frac{m_{ff}(t)}{\tau_{ff}}\right)] \]

where \( x_p \) represents the measured FAR, \( a \) and \( b \) are negative and positive known constants respectively, \( v_1 \) is the control variable representing fuel discharge entering the manifold, \( n \) is the fraction of the fuel that impinges on the wall, \( m_{ff} \) is the mass of the manifold fuel film, \([1 - n]\) is the non-impinging part of the injected fuel flow, \( \tau_{ff} \) is time constant for the first order evaporation model. For the purpose of simulation, we consider the numerical values \( a = -2 \) and \( b = 3 \), \( n \) and \( \tau_{ff} \), as in assumption 1, are bounded by \( 0 < n < 1 \) and \( 0 < \tau_{ff} < 2s \). The aim is to employ the proposed model with the designed control law to make the FAR \( x_r(t) \) track a desired value \( r = 1/15 = 0.0667 \). Figure (4) shows the time-response of system (1) by using control law (10). It is clear from this figure that the control law (10) achieves a good tracking performance and drives successfully the FAR variable to the desired value with a fast rate of convergence using a bounded controller \( v(t) \). It is observed from the figure that the same controller achieves the required objective for different values of \( n \) (0.05, 0.1, 0.15). As shown in this figure, the FAR is stably converging to the desired value \((1/15)\) with a good rate of convergence for the three values of \( n \) by using a bounded controller.

In a summery, the designed control law (10) provides a good robustness with respect to the evaporation time and fuel impinging the manifold wall. That is, controller (10) can ensure stability and convergence of FAR to the desired value in spite of the unknown disturbances represented by evaporation time and fuel impinging the manifold wall.
Figure (5). Time response of system (1) with three different values (0.05, 0.1, 0.15) for fuel fraction impinging manifold's wall using control law (10).
5 Conclusions and Scope for Future Work

A model for Fuel - Air Ratio in automobile spark ignition engine have been proposed. Also We consider the problem of controlling FAR for this model with accounts for both the injected fuel impinging on manifold walls and the fuel film evaporation. As shown in the proofs of Proposition 1 and Theorem 2 with its simulation in figures 3 & 4, the designed control law (10) can sufficiently solve the mentioned problem and successfully achieve stability and convergence of the present system reducing the steady-state error to zero with bounded controller despite the presence of unknown injected fuel impinging on manifold walls and the fuel film evaporation time. For future work, researchers can involve calibration of fuel delivery model and design of feed forward and feedback schemes for fuel delivery. The nonlinear controller should be designed based on the proposed identified model extended to full operating range of engine. Spark timing control is another area that can be addressed separately.

References


